Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

REVIEWS OF NEW WORKS.

Essays on Life Assurance. By EDWARD SANG. (*Read before the Society of Arts for Scotland,* 1839-40.)

MR. EDWARD SANG'S name is well known to actuaries in England as that of an author of a very elaborate collection of Life Assurance and Annuity Tables in illustration of the Carlisle rate of mortality. He is also almost equally well known in Scotland as the writer of the above Essays, which Mr. Wm. T. Thomson, of Edinburgh, who was in possession of the remaining copies, has kindly had gratuitously circulated among the members of the Institute of Actuaries in England, in consequence of some remarks at a recent Institute meeting at which that gentleman so efficiently presided. We have accordingly read and re-read Mr. Sang's Essays very carefully, and have thereby become convinced that what he wrote he also honestly believed; but, with every disposition to judge favourably, impartiality obliges us to think that, had Mr. Sang enjoyed the opportunities of a wider official experience, he would have considerably modified some of his notions, and abated some of his apparent desire to be showy and pungent, at the risk of obscuring better attributes. Every true mathematician must regret, for instance, such an offence against ripe scholarship as the introduction of the following piece of algebraical rhodomontade, as the only specimen of algebra throughout the Essays, merely to express the simple arithmetical notion that when we cannot fairly detect the law of any series of numbers, we are ostensibly compelled to use the numbers themselves.

> Taking a more general view of the subject than that which would be presented by the method of minimum squares, let ϕ denote an arbitrary function of the corrections whose sum is to be a minimum; put also ${}^0x$, ${}^1x$, ${}^2x$, &c., for the quantities actually observed, ${}^0X$, ${}^1X$, ${}^2X$ for the corrected ones; then would we have $\phi\,({}^0X - {}^0x) + \phi\,({}^1X - {}^1x) + \&c. = \text{min.}$ This condition of minimum resolves itself into the following equation:—
>
> $$\phi'\,(X - {}^0x)\,\delta\,{}^0X + \phi'\,({}^1X - {}^1x)\,\delta\,{}^1X, + \&c. = 0.$$
>
> Now here I observe, that if the nature of the law that ought to connect the quantities ${}^0X$, ${}^1X$, &c. be entirely unknown, that which should connect the variations $\delta\,{}^0X$, $\delta\,{}^1X$, &c. must be just as little known, the one being a mere sequence of the other, and that therefore, so far as our knowledge goes, these variations are entirely independent of each other; so that we must have the separate equations
>
> $$\phi'\,({}^0X - {}^0x) = 0, \quad \phi'\,({}^1X - {}^1x) = 0, \quad \&c.$$
>
> Assume, then, that from the nature of the arbitrary functions ϕ, $\phi' c = 0$, we shall have ${}^0X - {}^0x = c$, ${}^1X - {}^1x = c$, &c.; that is, all the quantities found by observation must be increased by a certain constant c, this constant being the error of position of the zero point of the scale by which the measurements are made. In our case there is no such error, and therefore we must have
>
> $${}^0X = x, \quad {}^1X = {}^1x, \ \&c.,$$
>
> so that, while working towards an unknown law, the condition of optimism requires that the actual results of the observations be used." (Pp. 3, 4, *Essay I., On the Law of Mortality.*)

Here, then, we have the differential calculus and the calculus of variations literally dragged in, not to usefully overcome some otherwise insurmountable difficulty, but, as we might almost suspect, for no better reason than to gratify the desire for a little display. It is the fairer to impute this false taste to Mr. Sang, because he had just before evinced a characteristic feature often combined with it, namely, a desire to sneer at great names. "*The* *method,*" he exclaims, "*of minimum squares of M. Legendre is arbitrary; why not minimum biquadrates? The whole matter is open to investigation; and, seizing the opportunity, I, &c.*" (P. 3.) Now Mr. Sang ought to have known, not merely that M. Legendre did not attempt to demonstrate the method of minimum squares, but that Gauss and La Place did attempt to do so, the former aided by the consideration of equi-probable errors, and the latter, with complete success, by that of equi-possible combinations.

Leaving the analytical path, Mr. Sang passes on, in his Second and Third Essays, to what may be called the mechanical view of life assurance matters. Towards this direction Mr. Sang's strength was principally exerted; and allow him his mechanical basis or set formula upon which to place his lever, and he might fairly essay, like a true disciple of Archimedes, to overturn the assurance world. But it is the very denial of this mechanical basis, of this *point d'appui*, that calls into exercise the profession of the actuary, and prevents the accountant or mechanician from stepping into his place. Popularly speaking, a table of mortality is supposed to be an exact picture of the events to come, and of course can be treated mechanically as such; but as expounders of the doctrine of probability, considerate actuaries should not forget its express and chief canon, that although $\frac{a}{b}$ may, *pro tanto*, be taken as the true chance of mortality for a single life, yet that it is not therefore even probable that out of b lives exactly a will die; and yet, in his Experimental Society, as displayed in these Essays, Mr. Sang has somewhat complacently taken the whole difficulty of deviation as solved by ignoring it, and supposed that in every year the *exact* number of deaths can be so efficiently predicated as to form set and stated elements in the annual progress of a commercial account-current. Had Mr. Sang really been the manager of an office, and dealt with business transactions in place of hypothetical ones, he would soon have had it forced on his attention that the laws of average were the only reigning laws in life assurance, and that it was no part of those laws that a society must of necessity always be in the state represented by the table of mortality, *pari passu*, but must openly be expected to deviate between certain extremes; and that it was therefore only by dealing with actual transactions, apart from theoretical loading, that the actual average could ever be really enunciated, because subject to differences in different societies, and therefore productive of different balance sheets. Indeed, the mere introduction of interest of money not only disturbs the popularly hypothetical state of things, but renders the average of so mixed a character, that as yet no definition has even been promulgated by theorists as to the relation existing between the expected deviations from a table of mortality and the consequent effects upon a table of annuities, and therefore upon a Life Assurance Company's valuations from year to year, even supposing the sums assured upon each life to be equal. We of course know, that if $d \pm x$ deaths be expected to follow $c \pm y$ deaths, that the average would not be disturbed by the substitution of $c \pm y$ deaths following $d \pm x$ deaths; but directly we employ even a constant, in place of a fluctuating, rate of interest of money, the whole state of the matter is changed, except in that solitary case in which $c \pm y$ and $d \pm x$ may happen to be equal.

Had, then, Mr. Sang devoted his thoughts and acquirements to the resolution and practical elucidation of such problems as these, he might indeed have claimed pre-eminence as a teacher on the subject, and as an originator of real novelties; but in the absence of these, our fear is, that however terse some of Mr. Sang's dicta may appear, and however fearless may be his apparent exposure of what he conceived to be fallacies, that his reputation in England will be rather weakened than strengthened by the general perusal of these Essays, considering how deservedly high he has been hitherto appreciated as the constructor of the tables already alluded to, and which are, indeed, at once a noble record of his industry, and an imperishable memento of what might have been expected from his calculative talents, had circumstances induced him to prosecute still further his intention of seriously adopting the actuarial profession, and thereby becoming imbued with those monetary tactics, without which no monetary transactions can ever be brought to a practically successful issue, dependent as such successful issues necessarily are, both upon the avoidance of exorbitance, exciting competition on the one hand, and theoretical rashness, weakening commercial credit on the other.

As examples of Mr. Sang's general style and notions, we freely insert the following copious quotations. The first is from Essay II., "On the Error of the Bonus system."

> Thus, both by precept and by example, is brought out the great primary objection to the bonus system—
>
> *That it enables the present members to appropriate to themselves all the over contributions of their predecessors, as well as to draw largely upon the contributions of their successors;* and it rather aggravates than palliates the evil, that, on account of the involved nature of the transaction, all this may be done unwittingly with the best intentions in the world.
>
> In the case of a Mutual Society, it is evidently proper that the interests of all classes should be alike, and that any arrangement which sets these interests in opposition to each other must be injurious to the welfare of the whole. To have shown, then, that the bonus system has this tendency, and that it gives to one class the control of the interests of other classes, and to have cited a single example of the actual operation of this evil bias, is sufficient, in my estimation, to condemn it, and to lead us to look out for some other arrangement free of such defects. (P. 20.)

The remainder are from Essay III., "On what constitute the Profits of a Mutual Assurance Society, and on the only Equitable Method of Distributing these among the Members."

> It is a singular fact, that, on a matter apparently so simple as the profits of a Life Assurance Society, and also so important as to be constantly before the minds of men of business, the most opposite ideas are entertained and acted on. Sums of money amounting to millions of pounds are actually distributed, not according to what is just and equitable in itself, but according to some mere opinion founded on some scheme for obviating one error by the commission of another.
>
> The crudest of these ideas, and that one which I must first discuss, is this—That the profit on any policy is the excess of the premiums paid over the sum assured; and the most absurd of its corollaries—That therefore no member who has not paid up, in premium and interest, the full sum assured upon his life, can be entitled to share in profits to which he has contributed nothing. (P. 21.)
>
> Such a notion can only have sprung up in a mind that had at the time forgotten the whole drift and intention of life assurance; and notwithstanding the high authority of the *Quarterly Review*, I fear not to assert that the maintaining of it betrays the most narrow and short sighted views of the subject.
>
> The very intention of a Life Assurance Society is to provide a sum for the heirs of those who die soon, out of the contributions of those who are fortunate enough to live long; and it sounds rather odd to say that the society sustains a loss by the accomplishment of the very purpose for which it was designed. If every one lived during the average time computed for him there would be no loss; and in the same way, were there never to be any fires, Fire Insurance Companies never would sustain a single loss—how wealthy they would soon become!!

The idea that those members of a Mutual Assurance Society through whom loss is sustained have no right to share in the profits, thus amounts, when fully carried out, to a dissolution of the Society, so far as a community of risk is concerned; and is, therefore, subversive of all the principles of assurance. " Oh ! but," exclaim the advocates of this principle, " we never intended to carry it to such an extent as you have done." Very true; and the reason is, that not having understood the nature of the principle, you never dreamed that it could be carried so far. But to take a milder case, I shall instance a Yearly Mutual Fire Insurance Society, who find at the end of the year the total receipt in premium greater than the total loss by fire, say double of it. Then is it not quite clear that if this could have been foreseen only half the premiums would have been exacted; and that had this half only been taken, those parties even through whom loss was sustained would have paid less; so that the surplus falls to be distributed among the whole of the policy-holders? In such a society there is established, for the time, a perfect community of property *quoad* fire, and the society has no business to inquire (in distributing either the profit or the loss) whether the damage have been done on the property of A or on that of B.

Let us imagine a Yearly Life Society, for simplicity having its members all of one age. At the commencement of the year each has paid his premium to assure £100, should he die within the year. At the end of the year it would be known whether a greater or less number of deaths had occurred than the expected number, and the overplus or deficiency would at once be known. It seems to me to be quite clear, that in case of an overplus, part would need to be returned to the heirs of the deceased members; just for the same reason, that if a deficiency had occurred these heirs would have had their proportion of that deficiency retained from the sum assured. If this principle hold in a *yearly* society, it is quite clear that it must extend also to a life one. (P. 22.)

Thus it has been usual to consider only those who have whole life assurances as entitled to share in the profit. I presume that this has not been arranged because there is any impropriety in a member till sixty, or during any limited term, receiving that share of the profit to which he is justly entitled, but because the method of computing his share has transcended the powers of the calculator.

The general nature of the question is stated in the following proposition:—

A number of persons form themselves into a society for assurance against certain contingencies of life. Some wish to secure a certain payment to their heirs in the event of their death, paying the price of it at once, in several terms, or annually during life; others wish to secure a sum to themselves should they attain a certain age, and so on.

To carry this scheme into effect, the prices of the various risks are computed, as near as may be, according to the best existing information, and some addition is made for the sake of security.

After a lapse of time the over payments will have accumulated, and will fall to be divided among the partners. (P. 40.)

The first branch of the question relates to the *time* at which the apportionment ought to take place. This must be determined by a reference to the nature of the transaction gone into.

Each member has agreed to pay a certain premium during the currency of his policy, or during part of that currency, on condition that he shall obtain a certain payment when the policy falls, and shall have a share in the profit or loss of the society. It seems to me perfectly clear that the agreement is to last during the whole currency of his policy, and that no member is entitled to claim his share of the profit until his policy expire. He is not entitled to draw out his share of the profit at any intermediate time.

This principle is implied in the scheme of declaring a bonus.

It would evidently be very unsafe to allow the existing members to draw out in ready money any of what might appear as surplus. This would be to go against the general tenor of the transactions, which tenor is, that the gross price shall remain deposited during the currency of the policy, that the overcharge contained in it may afford a guarantee for the sum assured. Were a payment in ready money to be made, how would the public look upon the transaction? Persons intending to become members would at once take the alarm; the rather so, if they observed parties who had entered only a few days before the division bringing off, as their *share* of profit, more than they had paid as premium. Such an arrangement would give full scope to the cupidity of the existing members, and would not bear the light. No one, say the lookers on, ought to have his share of the profits until the profits realized during the whole currency of his policy shall have been ascertained. The scheme of bonus, I repeat and insist upon, is just this arrangement concealed by a very flimsy veil, and rendered by that very disguise still more dangerous.

A body of men met in deliberation would hesitate before they put in their pockets a

> sum of ready money. They would consider over and again whether it were proper to take it; but a bonus, a sum *never to be touched by themselves*, does not awaken any such doubts, and although it did so, the resolution of those doubts is in many cases beyond their own power. Yet how very flimsy is the veil! In some societies the bonus may be surrendered, and the price of that surrender had in READY MONEY. The society dare not declare a ready money bonus; it proceeds to compute the equivalent contingent bonus, which contingent bonus it assigns, and then allows the members, if they choose, to dispose of that contingency. Were the sums of money so operated on trifling, this roundabout process would afford some amusement, the actual calculation bearing the strictest resemblance to those operations described in our elementary treatises on arithmetic as proofs. The flimsiness of this contrivance is remarkable, and has at once been detected by those north of the Tweed; their contrivance is one degree more knowing, and consists in allowing the members not to sell the contingency to the office, but to claim a corresponding deduction from their annual premiums. This certainly keeps the character of the transaction better out of view, but it does not change that character; nor, indeed, would that be changed by a refusal to compound in any way for the additional assurance.
>
> The real state of the agreement between parties is this—that each is willing to deposit a certain sum, regulated by the best information to be had, and agrees to abide the result of the best information to be obtained at the expiry of his policy. Not until the actual expiry of his policy, then, should any declaration be made of the share of the capital stock belonging to him. I shall be told of the *difficulty* of making such a valuation. I am, however, not avoiding, but seeking difficulties; no other first step toward the overcoming of them being known to me: the difficulties are there, whether we look at them or not. (P. 41.)

On referring to the "Select Papers read before the Society of Arts of Scotland" for the same period, we find Mr. Sang's Essays followed by a very excellent paper in opposition to them, read before the Society, 25th March, 1840, entitled "Remarks on the Profits of Life Assurance Societies, and the division thereof; with reference to certain statements contained in Mr. Sang's Essay on Life Assurance Societies. By William Wood."

Our want of space would prevent us entering upon Mr. Wood's paper, even if the consideration did not, that the general subject is one that has already been treated of more than once in our own pages.

Rational Arithmetic. By Mrs. G. R. PORTER. London: John Murray, Albemarle Street. 1852.

WE have much pleasure in calling attention to this little work, the professed object of which is, as the author states in the Preface, "to assist in rescuing arithmetic from the degraded rank it at present occupies among intellectual pursuits."

The author's aim is to show the *rationale* of the science, and not only to teach arithmetic, but at the same time to exercise the mental faculties of students; and, as she says in the words of Locke, "not so much to make them mathematicians as to make them reasonable creatures." To this end the author has avoided giving any rule without first having shown upon what principles the rule is founded.

Few writers have treated arithmetic on this plan; and while almost every scientific subject has been theoretically and well handled by many competent authors, we can scarcely name one, but De Morgan, who has departed from the old routine plan of the school treatises on arithmetic. This is a step in the right direction, and Mrs. Porter's work will prove a valuable first-book to the student in our profession—it will teach him to reflect, and bring forth and strengthen his reasoning powers, and enable